MIXTURE PROBLEMS – RADIATOR

❑ INTRODUCTION

Suppose a 12-liter radiator is filled with an antifreeze/water solution at a 25% concentration of antifreeze. In that case, 25% of the 12 liters is antifreeze, while the rest (75%) is water. Multiplying 12 liters by 25% shows that 3 liters of the solution are antifreeze, and the other 9 liters (12 liters × 75%) are water. Thus, the key formula we need for the percent mixture problems in this chapter is

Quantity × Concentration = Actual Amount

❑ EXAMPLE

A 10-liter radiator is filled with 42% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final solution up to 58%?

Solution: We'll set up all the info in a chart, utilizing the above formula:

	Quantity (in liters) ×	Concentration (of antifreeze) =	Actual Amount (of antifreeze)
Original	10	42%	4.2
Drain	x	42%	$0.42\,x$
Replace	x	100%	x
Final	10	58%	5.8

Notice that the four rows in our chart represent each of the problem's four phases.

In the statement of the problem, we are given that it's a 10-liter radiator filled with 42% antifreeze. The product of these two numbers is 4.2, as per the formula given above.

The question asks us how much to remove -- we don't know, so this is where the variable x comes in. What is the concentration of the x liters we're to remove? It's the same as the concentration of the entire 10 liters: 42%. And, of course, the product of x with 42% is $0.42x$.

Now for the 3rd row of the chart. The number of liters of pure antifreeze needed to refill the radiator must equal the number of liters drained off, namely x. And the refill fluid is pure antifreeze, which has a concentration of 100%; hence, the last entry in the row is $x \times 100\%$, or simply x.

The bottom row: The radiator has been refilled, so it contains 10 liters of solution, just as in the original row. The concentration of this final mixture is stipulated to be 58%, and the product is clearly 5.8.

Thus, looking at the right-hand column of the chart (Actual Amount), we can say that we <u>started</u> with 4.2 liters of antifreeze, <u>subtracted</u> $0.42x$ liters, <u>added</u> x liters, and <u>ended up</u> with 5.8 liters of antifreeze. Now we translate these thoughts into an equation:

$$4.2 - 0.42x + x = 5.8$$

$$\Rightarrow \quad 0.58x = 1.6$$

$$\Rightarrow \quad x = 2.76, \text{ and thus}$$

$$\boxed{\text{2.76 liters should be drained out}}$$

Note: If the antifreeze solution needs to be diluted, then we drain a certain amount of the solution and replace it with pure water, which has an antifreeze concentration of 0%.

Homework

1. A 9-liter radiator is filled with 67% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final mixture up to 95% antifreeze?

2. A 5-liter radiator is filled with 58% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure water to bring the concentration of the final mixture down to 25% antifreeze?

3. A 4-liter radiator is filled with 32% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure antifreeze to bring the concentration of the final mixture up to 75% antifreeze?

4. A 20-liter radiator is filled with 75% antifreeze solution. How much of the antifreeze solution must be drained and replaced with pure water to bring the concentration of the final mixture down to 23% antifreeze?

Solutions

1. 7.64 liters

2. Note: The concentration of antifreeze in pure water is 0%.
$5(.58) - .58x + 0x = 5(.25) \Rightarrow$ 2.84 liters

3. 2.53 liters

4. 13.87 liters